PHYSICAL REVIEW E **78**, 026407 (2008)

Friction-force model for Maxwellian drifting ions in weakly ionized plasmas

L. Conde,[1,*] L. F. Ibáñez,[1] and J. Lambás[2]
[1]Departamento Física Aplicada E.T.S.I. Aeronáuticos, Universidad Politécnica de Madrid, 28040 Madrid, Spain
[2]Departamento de Infraestructuras y Sistemas Aeroespaciales E.T.S.I. Aeronáuticos, Universidad Politécnica de Madrid, 28040 Madrid, Spain
(Received 24 March 2008; published 15 August 2008)

Analytical expressions for the collision frequency for momentum transfer and the friction force experienced by a Maxwellian ion population drifting with respect to the uniform neutral atom background are derived. The calculations make use of different models for the collision cross section for momentum transfer accounting for the relative speed v_r between the colliding particles. These results are compared with the currently used semi-empirical equations for the friction force and collision frequency in the fluid equations for weakly ionized plasmas. The kinetic model calculations are in agreement for suprathermal ion flows while they present discrepancies of orders of magnitude for subthermal ion drift speeds u_d. However, for the collision cross section $\sigma_m \sim 1/v_r$, the magnitude of the friction force results proportional to u_d and the collision frequency becomes constant regardless of the magnitude of the ion drift speed in both cases. These results are relevant for ion populations drifting in a plasma which could be approximated by shifted Maxwellian distributions, as in collisional plasma sheaths or plasma double layers.

DOI: 10.1103/PhysRevE.78.026407 PACS number(s): 52.25.Fi, 52.25.Ya, 52.20.Hv

I. INTRODUCTION

The collisional transference of energy and momentum from flowing ions to the neutral atom background are relevant processes in weakly ionized plasmas [1,2]. High energy groups of ions are observed in experiments [3–8] and different technological applications make use of this fact. The material modification techniques by ion implantation accelerate groups of ions to impact the target material in order to modify its surface properties [3–5]. Fast ions are always found in plasma double layers accelerated downstream by the plasma voltage drop with drift speeds higher than the ion thermal velocity [7,6]. The increments of energetic ions in the exhaust plasma plume of ion thrusters for space propulsion improve the thrust and operational efficiency of these devices [8].

These ions flow under large scale electric fields which develop into the bulk plasma and the elastic and charge exchange collisions convey momentum and energy from the accelerated ions to the neutral atoms. The motions of both species become coupled reaching to equal kinetic temperatures $K_BT_i \simeq K_BT_a = K_BT$ [1,2]. Therefore, these fast ion populations are often approximated by shifted Maxwellian distribution functions [3–6].

However, the kinetic description of these flowing ions requires solutions of the Boltzmann equation for the ion distribution function $f_i(\boldsymbol{v}_i,\boldsymbol{r},t)$ far from equilibrium,

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_i \cdot \boldsymbol{\nabla}_r\right) f_i + \frac{1}{m_i}\boldsymbol{\nabla}_v \cdot (\boldsymbol{F}_e f_i) = \left(\frac{\partial f_i}{\partial t}\right)_{col}, \tag{1}$$

where $(\partial f_i/\partial t)_{col}$ is the ion collision operator and $\boldsymbol{F}_e$ the external driving force. The macroscopic conservation laws for ion density, momentum and energy are obtained by taking moments of Eq. (1) [1,2].

In these fluid equations the fluid friction force $\boldsymbol{F}_{ia}$ between flowing ions and neutral atoms accounting from the collisional momentum transfer between both species is

$$\boldsymbol{F}_{ia} = - n_i n_a \mu_{ia} \langle \sigma_m(v_r)\boldsymbol{v}_r v_r\rangle. \tag{2}$$

The particle number densities are n_i and n_a and $\mu_{ia} = m_i m_a/(m_i+m_a)$ is the reduced mass [1,2]. The vector $\boldsymbol{I}_f = \langle \sigma_m(v_r)\boldsymbol{v}_r v_r\rangle$ is the average of the relative speed $\boldsymbol{v}_r = \boldsymbol{v}_i - \boldsymbol{v}_a$ of colliding particles, $|\boldsymbol{v}_r| = v_r$ and the momentum transfer cross section $\sigma_m(v_r)$ over the ion $f_i(\boldsymbol{v}_i,\boldsymbol{r},t)$ and neutral atoms $f_a(\boldsymbol{v}_a,\boldsymbol{r},t)$ energy distribution functions,

$$\boldsymbol{I}_f = \int \boldsymbol{v}_r v_r \sigma_m(v_r) f_a f_i d\boldsymbol{v}_i d\boldsymbol{v}_a. \tag{3}$$

In this equation, the total cross section $\sigma_m(v_r) = \sigma_{el}(v_r) + \sigma_{cx}(v_r)$ considers the contributions of both elastic $\sigma_{el}(v_r)$ and charge exchange $\sigma_{cx}(v_r)$ collisions [1,2].

However, due to the complexity of the collision operator, exact or even approximate solutions of the above Boltzmann equation are exceptionally rare. Therefore, approximate expressions are used for $\boldsymbol{F}_{ia}$, in nonequilibrium plasmas where the electron temperature is $K_BT_e \gg K_BT$, the relevant collisional speed is the ion sound velocity $c_s = \sqrt{2K_BT_e/\mu_{ia}}$ [2]. This latter is usually large compared with the drift velocity $\boldsymbol{u}_d = \boldsymbol{u}_i - \boldsymbol{u}_a$ between the ion $\boldsymbol{u}_i$ and neutral atom $\boldsymbol{u}_a$ fluid velocities. In this case, the cross section $\sigma_m(v_r)$ in Eq. (3) is currently approximated by a constant value $\sigma_c \simeq \sigma_m(c_s)$, leading to

$$\boldsymbol{F}_{ia} \simeq - n_i \mu_{ia} \nu_m (\boldsymbol{u}_i - \boldsymbol{u}_a),$$

where $\nu_m = n_a \sigma_c c_s$ is the constant collision frequency for momentum transfer [1,2].

This approximation apparently fails for large drift speeds, as in collisional sheaths or plasma double layers where ions are accelerated by potential drops larger than the electron thermal energy K_BT_e. Thus, for drift velocities of the order of

*luis.conde@upm.es

c_s the effective ion speed for collisions is the ion drift velocity $\boldsymbol{u}_d$ rather than the ion sound speed. Therefore, in order to account for either large or low values of $\boldsymbol{u}_d$, in Eq. (2), this latter is currently replaced by

$$\boldsymbol{F}_{ia}(k,u_d) = -n_i\mu_{ia}\nu_m(k,u_d)\boldsymbol{u}_d, \tag{4}$$

where $\nu_m(k,u_d)=n_a\sigma_{ia}(k,u_d)u_d$ is the collision frequency, and

$$\sigma_{ia}(k,u_d) = \sigma_o\left(\frac{c_s}{u_d}\right)^k. \tag{5}$$

The value for σ_o is determined from experimental data fitting of the momentum transfer cross section and $0\leqslant k\leqslant 2$ is a dimensionless parameter [9,10].

The above expression for $\sigma_{ia}(k,u_d)$ is suggested by the dependence of the experimental data with the relative speed v_r of colliding particles [2,11–15]. In the classical approximation $\sigma_m(v_r)\sim 1/v_r$ for collisions between ions and nonpolar neutrals, and $\sigma_m(v_r)\sim 1/v_r^2$ for polar atoms or molecules [11–15]. These cross sections are also consistent with the suggested models of $\sigma_m(v_r)$ for ion flux models in plasma sheaths [12] as well as with the experimental results for charge exchange and transport for ions [13,14]. The cross sections with $\sigma_{ia}(k,u_d)$ with $k=0$ and 1 are also employed in Ref. [15] to relate the collision properties with the stationary momentum distributions in weakly nonideal plasma of a stellar core.

The friction force $\boldsymbol{F}_{ia}(k,u_d)$ has been widely employed in modeling collisional plasma sheaths for both classical [9,10,16–20] and dusty plasmas [21,22]. The constant mean free path model [16,20] with the collision rate $\nu_m(u_d)=u_d/\lambda_i$, where $\lambda_i=1/(n_a\sigma_o)$ is the ion mean free path and is obtained with $k=0$. The constant collision frequency approximation [9,10,16–20,22] where $\nu_m\simeq n_a\sigma_o c_s$ is also recovered for $k=1$. Finally, the constant drag force model with $\boldsymbol{F}_{ia}=\mu_i n_i n_a c_s^2$ and $\nu_m(u_d)=n_a\sigma_o c_s^2/u_d$ is also deduced with $k=2$.

However, the calculation of the actual friction force and collision frequency between drifting species in a plasma requires of a kinetic approach [23–25,27–29]. In the following analysis, the ion distribution functions are modeled as shifted Maxwellians in order to derive analytic expressions for the macroscopic friction force and collision frequency of the fast ion groups observed in the experiments [3–8]. This approximation represents the lowest order contribution to the nonequilibrium single ion kinetic theory distribution of Eq. (1) [1,2].

To the best of our knowledge, no analytical expressions are available for the ion drag force and the elastic collision frequency of ions moving into a neutral gas for the cross sections $\sigma_m(k,u_d)$ for $k=0$, 1, and 2 using this approximation for $f_i(\boldsymbol{v}_i,\boldsymbol{r},t)$.

As we shall see, this model friction force is strongly dependent on the expression for the cross section. For drift speeds over the ion thermal velocity, the force $\boldsymbol{F}_{ia}(k,u_d)$ may differ by orders of magnitude in accordance to the value of k. The currently used Eqs. (4) and (5) with $k=0$ and 2 constitute a good approximation in the limit of drift speeds u_d higher than the ion thermal speed c_μ. However, the discrepancies with these kinetic calculations are of orders of magnitude for $u_d<c_\mu$. For the particular case of $k=1$, the friction force results proportional to the drift speed and the collision frequency also constant for all values of u_d.

II. THE FRICTION FORCE

The ion and neutral atom velocities are considered in the laboratory frame where the neutral atom background is at rest. The ion speed $\boldsymbol{v}_i=\boldsymbol{w}_i+\boldsymbol{u}_d$ is decomposed into a random component $\boldsymbol{w}_i$ and a constant drift speed $\boldsymbol{u}_d$. Then, $\boldsymbol{g}=\boldsymbol{v}_i-\boldsymbol{v}_a$ and $\boldsymbol{G}=(m_i\boldsymbol{v}_i+m_a\boldsymbol{v}_a)/M$ where $M=m_a+m_i$ are the relative and center of mass velocities of colliding particles.

The ion and neutral populations are considered with equal kinetic temperatures in accordance with the experimental evidence [3–8]. The electric field in the plasma which accelerates the ions is constant in time and points along a fixed direction in space, parallel to the drift speed $\boldsymbol{u}_d$.

In these conditions, the energy distribution $f_a(\boldsymbol{v}_a)$ of neutrals with speed $\boldsymbol{v}_a$ is a Maxwellian with temperature K_BT, while the velocity distribution function for ions could be approximated by

$$f_i(\boldsymbol{v}_i) = \frac{1}{\pi^{3/2}c_i^3}\exp\left(-\frac{(\boldsymbol{v}_i-\boldsymbol{u}_d)^2}{c_i^2}\right),$$

where $c_i=\sqrt{2K_BT/m_i}$ and $c_a=\sqrt{2K_BT/m_a}$ are the ion and neutral thermal speeds.

The average in Eq. (2), $\boldsymbol{I}_k(\boldsymbol{u}_d)=\langle\sigma_m(k,v_r)v_r\boldsymbol{v}_r\rangle$, is

$$\boldsymbol{I}_k(\boldsymbol{u}_d) = \frac{e^{-u_d^2/c_i^2}}{(\pi c_a c_i)^3}\int g\boldsymbol{g}\sigma_m(k,v_r) \times\exp\left(-\frac{m_iv_i^2+m_av_a^2-2\boldsymbol{v}_i\cdot\boldsymbol{u}_d}{2K_BT}\right)d\boldsymbol{v}_i d\boldsymbol{v}_a,$$

which is transformed into

$$\int g\boldsymbol{g}\sigma_m(n,g)\exp\left(-\frac{g^2+2\boldsymbol{g}\cdot\boldsymbol{u}_d}{c_\mu^2}\right) \times\exp\left(-\frac{G^2}{c_m^2}+\frac{2\boldsymbol{G}\cdot\boldsymbol{u}_d}{c_i^2}\right)d\boldsymbol{G}d\boldsymbol{g}$$

where $c_m=\sqrt{2K_BT/M}$ and $c_\mu=\sqrt{2K_BT/\mu_{ia}}$. The drift speed $\boldsymbol{u}_d$ points along a fixed direction and therefore $\boldsymbol{u}_d\cdot\boldsymbol{G}=u_dG\cos\theta$, where θ represents the angle between $\boldsymbol{G}$ and $\boldsymbol{u}_d$. The integral over $\boldsymbol{G}$ is

$$I_G(u_d) = 2\pi\int_0^{+\infty}\exp\left(-\frac{G^2}{c_m^2}\right)G^2dG \times\int_0^{\pi}\exp\left(\frac{2Gu_d\cos\theta}{c_i^2}\right)\sin\theta d\theta,$$

and integrating

$$I_G(u_d) = \frac{2\pi c_i^2}{u_d}\int_0^{+\infty}\sinh\left(\frac{2u_dG}{c_i^2}\right)G\exp\left(-\frac{G^2}{c_m^2}\right)dG.$$

We obtain

$$I_G(u_d) = \pi^{3/2} c_m^3 \exp\left(u_d^2 \frac{c_m^2}{c_i^4}\right),$$

and the average $\boldsymbol{I}_k(u_d)$ reads

$$\boldsymbol{I}_k(u_d) = \frac{e^{-u_d^2/c_\mu^2}}{(\sqrt{\pi}c_\mu)^3}\boldsymbol{P}_k(u_d), \tag{6}$$

where

$$\boldsymbol{P}_k(u_d) = \int \sigma_m(k,g) g \boldsymbol{g} e^{-(g/c_\mu)^2} e^{-(\boldsymbol{g}\cdot\boldsymbol{u}_d)/c_\mu^2} d\boldsymbol{g}.$$

Because of the symmetry, the only contribution to $\boldsymbol{P}_k(u_d)$ lies along the direction parallel to the drift speed $\boldsymbol{u}_d$. As before, $\boldsymbol{g}\cdot\boldsymbol{u}_d = g u_d \cos\phi$ where ϕ is now the angle between $\boldsymbol{g}$ and $\boldsymbol{u}_d$ and therefore,

$$\boldsymbol{P}_k(u_d) = 2\pi \hat{\boldsymbol{u}}_d \int_0^{+\infty} \sigma_m(k,g) g^4 e^{-(g/c_\mu)^2} dg \times \int_0^{\pi} \exp\left(-\frac{2 g u_d \cos\phi}{c_\mu^2}\right) \cos\phi \sin\phi d\phi,$$

where $\hat{\boldsymbol{u}}_d = \boldsymbol{u}_d/u_d$ is the unit vector along the direction of the drift. After the integration, we obtain

$$\boldsymbol{P}_k(u_d) = 2\pi \hat{\boldsymbol{u}}_d \int_0^{\infty} \sigma_m(k,g) g^4 e^{-(g/c_\mu)^2} \times \left[\frac{c_\mu^2}{g u_d}\cosh\left(\frac{2 g u_d}{c_\mu^2}\right) - \frac{c_\mu^4}{g^2 u_d^2}\sinh\left(\frac{2 g u_d}{c_\mu^2}\right)\right] dg$$

In Refs. [25,26] a similar equation was deduced for the drag force between dust particles and flowing ions in a dusty plasma. The values of the friction force were obtained by the numerical evaluation of the integral because of the more involved expression for the collision cross section between dust and ions.

Introducing the dimensionless relative $\tilde{g} = g/c_\mu$ and drift $\tilde{u}_d = u_d/c_\mu$ velocities, Eq. (6) becomes

$$\boldsymbol{I}_k(\tilde{u}_d) = \hat{\boldsymbol{u}}_d \frac{c_\mu}{\sqrt{\pi}} \frac{e^{-\tilde{u}^2}}{\tilde{u}} \int_0^{\infty} \sigma_m(k, c_\mu \tilde{g}) \times e^{-\tilde{g}^2} \tilde{g}^3 [S(\tilde{u}_d, \tilde{g}) + S(-\tilde{u}_d, \tilde{g})] d\tilde{g}, \tag{7}$$

where

$$S(\tilde{g}, \tilde{u}_d) = \left(1 - \frac{1}{\tilde{g}\tilde{u}_d}\right) e^{2\tilde{g}\tilde{u}_d}.$$

Finally, we introduce in Eq. (7) the expression for the total cross section suggested by Eq. (5),

$$\sigma_m(k, \tilde{g}) = \left(\frac{c_s}{c_\mu}\right)^k \frac{\sigma_o}{\tilde{g}^k}. \tag{8}$$

The expression for the friction force between the ion and neutral atom populations is

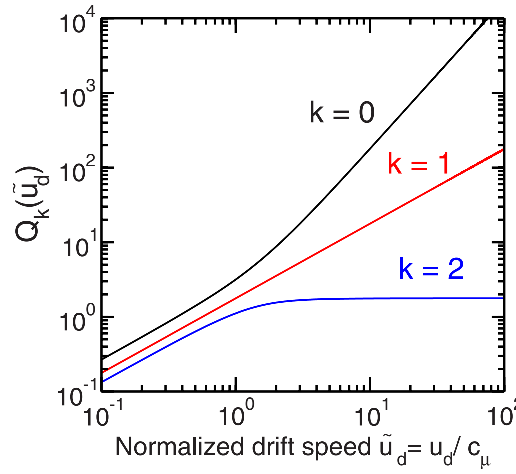

FIG. 1. (Color online) The function $Q_k(\tilde{u}_d)$ proportional to the magnitude of the friction force $\boldsymbol{H}_{ia}(k,\tilde{u}_d)$ for the cross sections given by Eq. (8) with $k=1$, 2, and 3.

$$\boldsymbol{H}_{ia}(k,\tilde{u}_d) = -n_a n_i \mu_{ia} \frac{\sigma_o}{\sqrt{\pi}} \frac{c_s^k}{c_\mu^{(k-2)}} Q_k(\tilde{u}_d) \hat{\boldsymbol{u}}_d. \tag{9}$$

The magnitude of the friction force $\boldsymbol{H}_{ia}(k,\tilde{u}_d)$ is determined by the functions $Q_k(\tilde{u}_d)$ which are odd functions of $\tilde{u}_d$. The constant cross section model corresponding to $k=0$ in Eq. (8) leads to

$$Q_o(\tilde{u}_d) = \frac{1}{4\tilde{u}_d^2}[2\tilde{u}_d(1+2\tilde{u}_d^2)e^{-\tilde{u}_d^2} + (4\tilde{u}_d^4 + 4\tilde{u}_d^2 - 1)\sqrt{\pi}\,\mathrm{erf}(\tilde{u}_d)], \tag{10}$$

where $\mathrm{erf}(\tilde{u}_d)$ is the error function. For the collision cross section between ions and nonpolar atoms ($k=1$), we obtain

$$Q_1(\tilde{u}_d) = \sqrt{\pi}\tilde{u}_d. \tag{11}$$

Finally, setting $k=2$ in Eq. (8), for the collision cross section model for ion and polar molecules,

$$Q_2(\tilde{u}_d) = \frac{e^{-\tilde{u}_d^2}}{\tilde{u}_d} + \sqrt{\pi}\left(1 - \frac{1}{2\tilde{u}_d^2}\right)\mathrm{erf}(\tilde{u}_d). \tag{12}$$

These functions $Q_k(\tilde{u})$ are represented in Fig. 1 for the different values of k.

III. THE ELASTIC COLLISION FREQUENCY

The elastic collision frequency for momentum transfer between ions and neutral atoms [1,2] is $f_m(k,u_d) = n_a n_i C_k(u_d)$ where the collision rate $C_k(u_d)$ is given by

$$C_k(u_d) = \int \sigma_m(k, v_r) v_r f_a(\boldsymbol{v}_a) f_i(\boldsymbol{v}_i) d\boldsymbol{v}_i d\boldsymbol{v}_a.$$

Using the center of mass $\boldsymbol{G}$ and relative $\boldsymbol{g}$ velocities and following the steps of Sec. II, the integration over $\boldsymbol{G}$ leads to a scalar equation similar to Eq. (6),

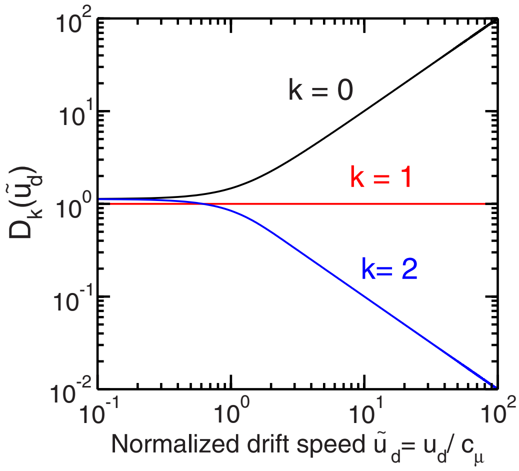

FIG. 2. (Color online) The function $D_k(\tilde{u}_d)$ proportional to the elastic collision frequency $f_m(k,\tilde{u}_d)$ for the cross sections given by Eq. (8) with $k=1,2$, and 3.

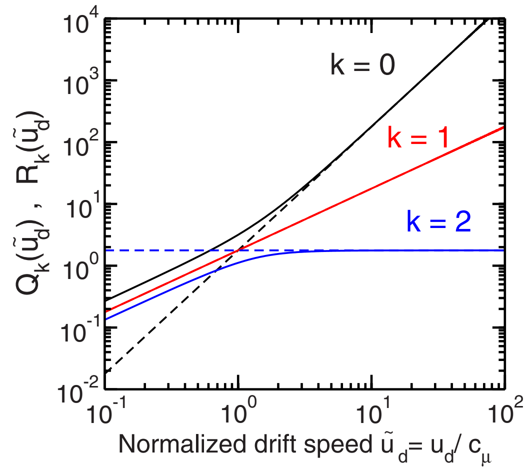

FIG. 3. (Color online) The magnitude $Q_k(\tilde{u}_d)$ of the friction force given by Eqs. (10)–(12) (solid lines) compared with the values of $R_k(\tilde{u}_d)$ of Eq. (17) (dotted lines).

$$C_k(u_d) = \frac{e^{-(u_d/c_\mu)^2}}{\pi^{3/2} c_\mu^3} \int \sigma_m(k,g) g \exp\left(-\frac{g^2 - 2\boldsymbol{g}\cdot\boldsymbol{u}_d}{c_\mu^2}\right) d\boldsymbol{g}.$$

After the integration over the angle ϕ between $\boldsymbol{g}$ and $\boldsymbol{u}_d$ and introducing the dimensionless speeds $\tilde{g}$ and $\tilde{u}_d$ as before,

$$C_k(u_d) = c_\mu \frac{2}{\sqrt{\pi}} \frac{e^{-\tilde{u}_d^2}}{\tilde{u}_d} \int_0^\infty \sigma_m(k,\tilde{g}) \tilde{g}^2 e^{-\tilde{g}^2} \sinh(2\tilde{u}_d \tilde{g}) d\tilde{g}.$$

The elastic collision frequencies for momentum transfer $f_m(k,\tilde{u}_d)$ are calculated from this last equation using the cross section models $\sigma_m(k,\tilde{g})$ of Eq. (8),

$$f_m(k,\tilde{u}_d) = n_a n_i \sigma_o \frac{c_s^k}{c_\mu^{(k-1)}} D_k(\tilde{u}_d). \tag{13}$$

The even functions $D_k(\tilde{u}_d)$ of the dimensionless drift speed are positive and the result for $k=0$ is

$$D_o(\tilde{u}_d) = \frac{e^{-\tilde{u}_d^2}}{\sqrt{\pi}} + \left(\tilde{u}_d + \frac{1}{2\tilde{u}_d}\right) \mathrm{erf}(\tilde{u}_d). \tag{14}$$

For the collision cross section between ions and nonpolar neutrals, $D_1(\tilde{u}_d)=1$, leading to a constant collision frequency $f_m(1,\tilde{u}_d)=\sigma_o n_a n_i c_s$ independent of the dimensionless drift speed. Finally, for $k=2$,

$$D_2(\tilde{u}_d) = \frac{\mathrm{erf}(\tilde{u}_d)}{\tilde{u}_d}. \tag{15}$$

Again, the magnitude of the collision frequency is determined by the functions $D_k(\tilde{u}_d)$ which are represented in Fig. 2.

IV. DISCUSSION

From Fig. 1 it is deduced that the different cross section models $\sigma_m(k,\tilde{u}_d)$ of Eq. (8) give for $\tilde{u}_d \gg 1$ values of $\boldsymbol{H}_{ia}(k,\tilde{u}_d)$ different by orders of magnitude. In the opposite limit of low drift speeds $Q_o \simeq 8\tilde{u}_d/3$ and $Q_2 \simeq 4\tilde{u}_d/3$, the magnitude of the friction forces become proportional $\tilde{\boldsymbol{u}}_d$ in all cases.

However, for $k=1$ corresponding to the collision cross section $\sigma_m(v_r) \sim 1/v_r$, we obtain $Q_1(\tilde{u}_d) = \sqrt{\pi}\tilde{u}_d$ regardless of the value of the drift speed. In this case the friction force results are proportional to $\tilde{u}_d$ for either suprathermal and subthermal ion flows.

In order to compare $\boldsymbol{H}_{ia}(k,\tilde{u}_d)$ of Eq. (9) with the friction force predicted by Eq. (4) this latter is rewritten as

$$\boldsymbol{F}_{ia}(k,\tilde{u}_d) = -n_a n_i \mu_{ai} \frac{\sigma_o}{\sqrt{\pi}} \frac{c_s^k}{c_\mu^{(k-2)}} R_k(\tilde{u}_d) \hat{\boldsymbol{u}}_d, \tag{16}$$

where

$$R_k(\tilde{u}_d) = \sqrt{\pi} \tilde{u}_d^{(2-k)}, \tag{17}$$

and these functions are represented together with $Q_k(\tilde{u}_d)$ in Fig. 3.

Thus, it is apparent that $\boldsymbol{F}_{ia}(k,\tilde{u}_d)$ given by Eqs. (16) and (17) is the limit for suprathermal ion flows of $\boldsymbol{H}_{ia}(k,\tilde{u}_d)$ and as in Eq. (17), in the limit of large drift speeds $Q_o(\tilde{u}_d)/\tilde{u}_d^2 \sim \sqrt{\pi}$ and $Q_2(\tilde{u}_d) \sim \sqrt{\pi}$.

However, for subthermal drift speeds the values predicted by $\boldsymbol{F}_{ia}(k,\tilde{u}_d)$ with $k=0$ and $k=2$ are rather different from those given by Eq. (9). The constant cross section model ($k=0$) underestimates in Eq. (16) the friction force for $\tilde{u}_d < 1$, while for the cross section $\sigma_m(v_r) \sim 1/v_r^2$ the values of $R_2(\tilde{u}_d)$ lie about two orders of magnitude over those of $Q_2(\tilde{u}_d)$. Nevertheless, for $\sigma_m(v_r) \sim 1/v_r$ corresponding to $k=1$, the friction forces predicted by Eqs. (16) and (9) are identical for all values of $\tilde{u}_d$.

The collision frequency $\nu_m(k,u_d)$ from Eqs. (4) and (5) can be expressed as

$$\nu_m(k,\tilde{u}_d) = n_a \sigma_o \frac{c_s^k}{c_\mu^{(k-1)}} S_k(\tilde{u}_d), \tag{18}$$

where

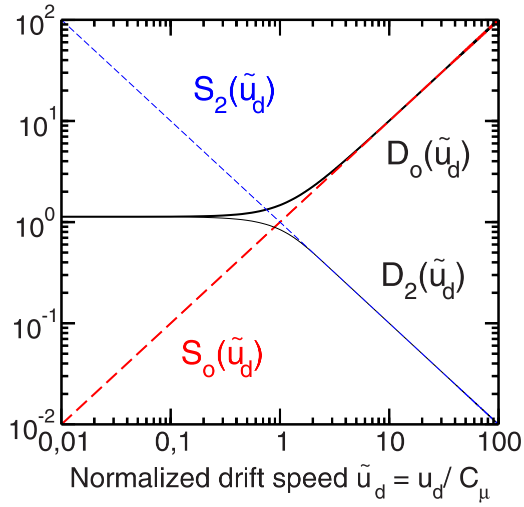

FIG. 4. (Color online) The magnitude the collision frequencies $D_k(\hat{u}_d)$ using Eqs. (14) and (15) compared with $R_k(\hat{u}_d)$ predicted by Eq. (19) with $k=0$ and 2.

$$S_k(\tilde{u}_d) = \tilde{u}^{(1-k)}, \tag{19}$$

which are compared in Fig. 4 with the functions $D_k(\tilde{u}_d)$ for $k=0$ and 2.

For $\sigma_m(v_r) \sim \sigma_c$ constant ($k=0$), the collision frequency grows with $\tilde{u}_d$ in Fig. 2 in agreement with the observed increment of the friction force in Fig. 1. On the contrary, for $k=2$, the collision frequency decreases for $\tilde{u}_d \gg 1$ in Fig. 2 and the friction force leads to a limit value in Fig. 1. While the amount of momentum transferred in each collision increments with the drift speed, the number of collisional events for $\tilde{u}_d \gg 1$ decreases, leading to small increments in the friction force. Finally, for the cross section model $\sigma_m(v_r) \sim 1/v_r$, the collision frequencies given by Eqs. (18) and (13) are equal to those for the friction force. Both are constant and independent of the relative drift speed.

Therefore, $\nu_m(k,\tilde{u}_d)$ is again a good approximation of $f_m(k,\tilde{u}_d)$ with $k=0$ and $k=2$ for suprathermal flows and in the limit of large $\tilde{u}_d$ we obtain $D_o(\tilde{u}_d) \sim \tilde{u}_d$ and $D_2(\tilde{u}_d) \sim 1/\tilde{u}_d$ as in Eq. (19).

On the contrary, the functions $S_o(\tilde{u}_d)$ and $S_2(\tilde{u}_d)$ differ by orders of magnitude from $D_o(\tilde{u}_d)$ and $D_2(\tilde{u}_d)$ for $\tilde{u}_d < 1$. In addition, for $\tilde{u}_d \sim 0$ the collision frequency reaches an equilibrium limit value. As is evidenced in Fig. 4, for low drift speeds we obtain $D_o(\tilde{u}_d) = D_2(\tilde{u}_d) \simeq 2/\sqrt{\pi}$, but this is not the case for the collision frequencies $\nu_m(k,\tilde{u}_d)$ calculated using Eq. (19) with $k=0$ and 2.

V. CONCLUSIONS

For large collision frequencies between ions and neutral atoms both populations of particles are in thermal equilibrium and the ion velocity distribution function could be approximated by a drifting Maxwellian [1,29]. This is the case of collisional plasma sheaths or double layers where ions flow in the plasma with respect to the neutral atom background driven by electric fields which are extended along many collisional mean free paths [6,29].

In these conditions, Figs. 1 and 2 evidence that the friction force $\boldsymbol{H}_{ia}(k,\tilde{u}_d)$ between both species is strongly dependent of the elastic cross section model for momentum transfer $\sigma_m(k,\tilde{u}_d)$. The results for the different cross sections are similar for subthermal flows while the values for either the collision frequency and friction forces apparently differ for $u_d > c_\mu$.

From Figs. 3 and 4 it is deduced that the current used for the drift dependent friction forces and collision frequencies of Eqs. (4) and (5) agree with the limit for suprathermal ion drift speeds of Eqs. (9) and (13), while for subthermal flows $u_d/c_\mu < 1$ the discrepancies are of orders of magnitude.

Finally, for the particular case of the collision cross section $\sigma_m(v_r) \sim 1/v_r$, which is in agreement with the experimental data [12,13], the friction force results proportional to u_d and the collision frequency constant for either suprathermal and subthermal flows.

These results are also valid for other colliding species where the elastic collision cross section models of Eq. (5) apply.

ACKNOWLEDGMENTS

The authors are grateful to Professor J.M. Donoso for stimulating discussions and the critical reading of the manuscript. This paper is supported by MEC of Spain under Grants ENE-2007-67406-C02-01 and HU2005-0022.

[1] L. C. Woods, *An Introduction to the Kinetic Theory of Gases and Magnetoplasmas* (Oxford University Press, Oxford, 1993), Chaps. 4 and 6.
[2] Y. P. Raizer, *Gas Discharge Physics* (Springer Verlag, Berlin, 1991), Chap. 2.
[3] Y.-S. Cho, H. J. Lee, C.-H. Park, H.-J. Lee, and S.-K. Lee, J. Appl. Phys. **100**, 123303 (2006).
[4] A. Anders and Efim Oks, J. Appl. Phys. **101**, 043304 (2007); Johanna Rosén, Jochen M. Schneider, and A. Anders, Appl. Phys. Lett. **89**, 141502 (2006).
[5] S. Amoruso, V. Berardi, R. Bruzzese, N. Spinelli, and X. Wang, Appl. Surf. Sci. **127-129**, 953 (1998).
[6] C. Charles, Plasma Sources Sci. Technol. **16**, R1 (2007).
[7] O. Sutherland, C. Charles, N. Plihon, and R. W. Boswell, Phys. Rev. Lett. **95**, 205002 (2005).
[8] A. Smirnov, Y. Raitses, and N. J. Fisch, Phys. Plasmas **14**, 057106 (2007).
[9] T. E. Sheridan and J. Goree, Phys. Fluids B **3**(10), 2796 (1991).
[10] J-Y. Liu, Z. X. Wang, and X. Wang, Phys. Plasmas **10**, 3032 (2003).
[11] G. G. Chernyi, A. A. Losev, S. O. Macheret, and B. V. Potapkin, *Physical and Chemical Processes in Gas Dynamics: Cross Sections and Rate Constants* (AIAA, Reston, 2002), Vol. 1 and

Vol. 196, Chap. 2.
[12] A. V. Phelps, J. Appl. Phys. **76**(2), 747 (1994).
[13] S. B. Radovanov, R. J. Van Brunt, J. K. Olthoff, and B. M. Jelenkovic, Phys. Rev. E **51**(6), 6036 (1995).
[14] M. V. V. S. Rao, R. J. Van Brunt, and J. K. Olthoff, Phys. Rev. E **54**(5), 5641 (1996).
[15] F. Ferro and P. Quarati, Phys. Rev. E **71**, 026408 (2005).
[16] K. U. Riemann, Phys. Plasmas **4**, 4158 (1997).
[17] R. N. Franklin and J. Snell, Phys. Plasmas **7**, 3077 (2000).
[18] X. P. Chen, Phys. Plasmas **5**, 804 (1998).
[19] S.-B. Wang and A. E. Wendt, IEEE Trans. Plasma Sci. **25**, 1358 (1999).
[20] M. S. Benilov, Phys. Plasmas **7**, 4403 (2000).
[21] S. V. Vladimirov and N. F. Cramer, Phys. Rev. E **62**, 2754 (2000).
[22] D. P. Resendes, G. Sorasio, and P. K. Shukla, Phys. Plasmas **9**, 2988 (2002).
[23] A. V. Ivlev, S. A. Khrapak, S. K. Zhdanov, G. E. Morfill, and G. Joyce, Phys. Rev. Lett. **92**, 205007 (2004).
[24] A. V. Ivlev, S. K. Zhdanov, S. A. Khrapak, and G. E. Morfill, Phys. Rev. E **71**, 016405 (2005).
[25] I. Denysenko, M. Y. Yu, L. Stenflo, and S. Xu, Phys. Rev. E **72**, 016405 (2005).
[26] J. E. Daugherty and B. Graves, J. Appl. Phys. **78**, 2279 (1995); J. Perrin, P. Molinas-Mata and P. Belenger, J. Phys. D **27**, 2499 (1994).
[27] V. Yaroshenko, S. Ratynskaia,S. Krapak,M. H. Thomas, M. Kretschmer, H. Höfner, G. E. Morfill, A. Zobnin, A. Usachev, O. Petrov, and P. Fortov,Phys. Plasmas **12**, 093503 (2005).
[28] J. Horwitz and P. M. Banks, Planet. Space Sci. **21**, 1975 (1973).
[29] L. Conde, L. F. Ibáñez, and C. Ferro-Fontán, Phys. Rev. E **64**, 046402 (2001).